Slide 1

11.3 Weighted Voting Systems

Rank any number of options in your order of preference.		Rate each between -10 and 10	
	Joe Smith	7	Joe Smith
1	John Citizen	10	John Citizen
3	Jane Doe	-3	Jane Doe
	Fred Rubble	0	Fred Rubble
2	Mary Hill	10	Mary Hill

1

Slide 2

There are different voting systems to the ones we've looked at.

Instead of focusing on the candidates, let's focus on the voters.

2

Slide 3

In a **weighted voting system**, the votes of some voters matters more than others.

Here, we will not have a "one person, one vote" principle.

Example: a stockholder with more shares has more of an effect on corporate policy than a stockholder with fewer shares.
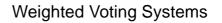
3

Slide 4

The **weight** of a voter is the number of votes they have for an issue.

A **quota** of votes is the number of votes need to get an issue passed.

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Slide 5

DEFINITIONS A **weighted voting system** with n voters is described by a set of numbers that are listed in the following format:
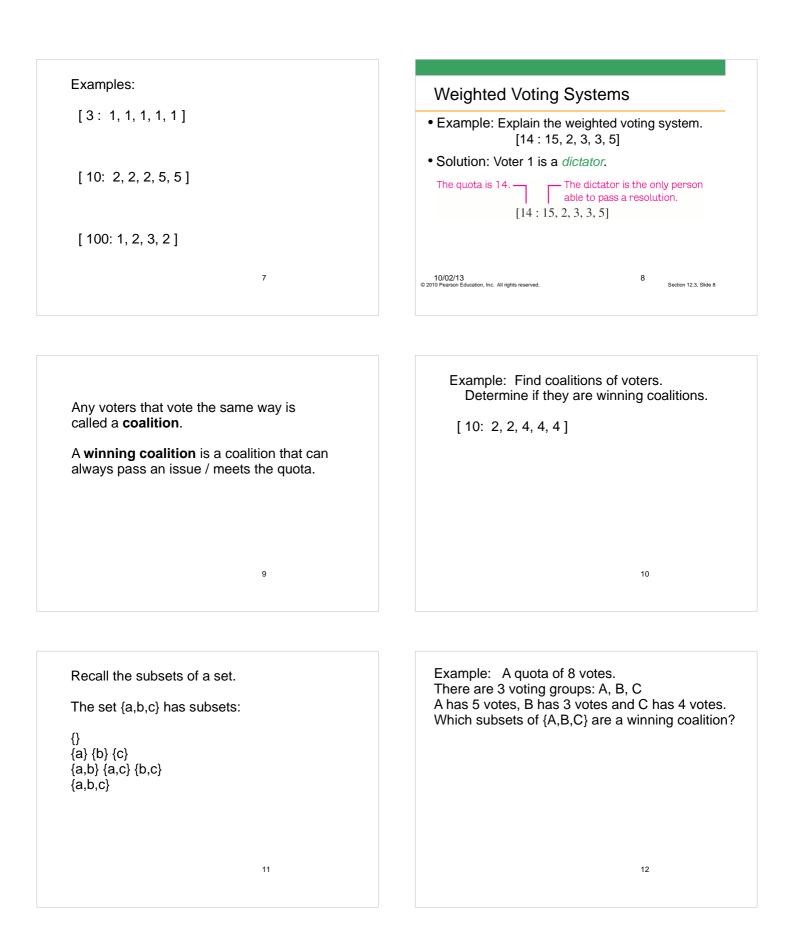
[quota: weight of voter 1, weight of voter 2, . . . , weight of voter n]

The **quota** is the number of votes necessary in this system to get a resolution passed. The numbers that follow, called **weights**, are the amount of votes controlled by voter 1, voter 2, etc.

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Slide 6

Weighted Voting Systems

• Example: Explain the weighted voting system.
[51 : 26, 26, 12, 12, 12, 12]

• Solution: The following diagram describes how to interpret this system.

[51 : 26, 26, 12, 12, 12, 12]

Need 51 votes to pass a resolution.

A and B each have 26 votes.

C, D, E, and F have 12 votes each.

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Examples:

[3 : 1, 1, 1, 1, 1]

[10: 2, 2, 2, 5, 5]

[100: 1, 2, 3, 2]

7

- Example: Explain the weighted voting system.
  [14 : 15, 2, 3, 3, 5]

- Solution: Voter 1 is a *dictator*.

The quota is 14. ⌐   ⌐ The dictator is the only person
                              able to pass a resolution.
            [14 : 15, 2, 3, 3, 5]

Any voters that vote the same way is called a **coalition**.

A **winning coalition** is a coalition that can always pass an issue / meets the quota.

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Example: Find coalitions of voters.
    Determine if they are winning coalitions.

[10: 2, 2, 4, 4, 4]

10

Recall the subsets of a set.

The set {a,b,c} has subsets:

{}
{a} {b} {c}
{a,b} {a,c} {b,c}
{a,b,c}

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Example: A quota of 8 votes.
There are 3 voting groups: A, B, C
A has 5 votes, B has 3 votes and C has 4 votes.
Which subsets of {A,B,C} are a winning coalition?

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Slide 13

Example: A quota of 8 votes.
There are 3 voting groups: A, B, C
A has 5 votes, B has 3 votes and C has 4 votes.
Which subsets of {A,B,C} are a winning coalition?

{A}	5	
{B}	3	
{C}	4	
{A,B}	8	winning
{A,C}	9	winning
{B,C}	7	
{A,B,C}	12	winning

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Slide 14

14

Slide 15

Example: Who is critical to get 8 votes?

{A}	5	
{B}	3	
{C}	4	
{A,B}	8	winning
{A,C}	9	winning
{B,C}	7	
{A,B,C}	12	winning

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Example: Who is critical to get 8 votes?

			critical
{A}	5		
{B}	3		
{C}	4		
{A,B}	8	winning	A, B
{A,C}	9	winning	A, C
{B,C}	7		
{A,B,C}	12	winning	A (more than 8 votes if B or C removed)

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Slide 17

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Slide 18

Compute the Banzhaf Power Index for A, B, C.

			critical
{A}	5		
{B}	3		
{C}	4		
{A,B}	8	winning	A, B
{A,C}	9	winning	A, C
{B,C}	7		
{A,B,C}	12	winning	A

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Compute the Banzhaf Power Index for A, B, C.

                              critical
{A}   5
{B}   3
{C}   4
{A,B}   8   winning   A, B   (Note: total
{A,C}   9   winning   A, C   critical voters
{B,C}   7   is 5 = 3+1+1)
{A,B,C} 12   winning   A

A critical 3 times, B critical 1 time, C critical 1 time

Banzhaf Power Index
A : 3/(3+1+1)   B : 1/(3+1+1)   C : 1/(3+1+1)

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Example: Quota of 10
Weights: R 9, D 8, I 3

Coalition	Weight		Critical Voters
{R}	9		
{D}	8		
{I}	3		
{R, D}	17	Winning	R, D
{R, I}	12	Winning	R, I
{D, I}	11	Winning	D, I
{R, D, I}	20	Winning	none

Remove any of these voters and the coalition no longer wins.

$$\frac{\text{the number of times R is critical in winning coalitions}}{\text{the total number of times voters are critical in winning coalitions}} = \frac{2}{6} = \frac{1}{3}.$$

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Q is a senior partner.   R,S,T are all associates.
To pass a motion, a senior partner and at least
2 associates need to vote in favor.
Find all coalitions and the BPI for each.

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Q is a senior partner.   R,S,T are all associates.
To pass a motion, a senior partner and at least
2 associates need to vote in favor.
Find all coalitions and the BPI for each.

Winning Coalitions:
{ Q, R, S }   { Q, R, T }   { Q, S, T }   {Q, R, S, T }

Critical Voters:
  Q, R, S   Q, R, T   Q, S, T   Q

Total critical voters in winning coalitions: 10

Banzhaf Power Index:
  Q : 4/10   R : 2/10   S : 2/10   T : 2/10

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The Banzhaf Power Index

• Example: A law has two senior partners
(Krooks and Cheatum) and four associates (W,
X, Y, and Z). To change any major policy of the
firm, Krooks, Cheatum, and at least two
associates must vote for the change. Calculate
the Banzhaf power index for each member of
this firm.

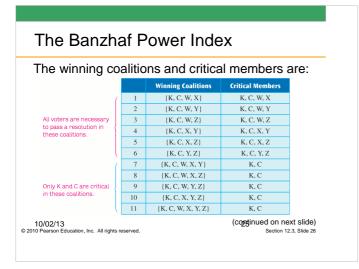• Need K and C, need at least 2 of W, X, Y, and Z

(continued on next slide)
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The Banzhaf Power Index

• Solution: We use {K, C, W, X, Y, Z} to
represent the firm. Since every winning coalition
includes {K, C} and any two of the other
associates, we only need to determine the
subsets of {W, X, Y, Z} with two or more
members to determine the winning coalitions.

2-Element Subsets of {W, X, Y, Z}	3-Element Subsets of {W, X, Y, Z}	4-Element Subsets of {W, X, Y, Z}
{W, X}, {W, Y}, {W, Z} {X, Y}, {X, Z}, {Y, Z}	{W, X, Y}, {W, X, Z}, {W, Y, Z}, {X, Y, Z}	{W, X, Y, Z}

(continued on next slide)
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The Banzhaf Power Index

The winning coalitions and critical members are:

	Winning Coalitions	Critical Members
1	{K, C, W, X}	K, C, W, X
2	{K, C, W, Y}	K, C, W, Y
3	{K, C, W, Z}	K, C, W, Z
4	{K, C, X, Y}	K, C, X, Y
5	{K, C, X, Z}	K, C, X, Z
6	{K, C, Y, Z}	K, C, Y, Z
7	{K, C, W, X, Y}	K, C
8	{K, C, W, X, Z}	K, C
9	{K, C, W, Y, Z}	K, C
10	{K, C, X, Y, Z}	K, C
11	{K, C, W, X, Y, Z}	K, C

All voters are necessary to pass a resolution in these coalitions. (coalitions 1–6)

Only K and C are critical in these coalitions. (coalitions 7–11)

(continued on next slide)

The Banzhaf Power Index

K and C are critical members 11 times, whereas W, X, Y, and Z are each critical members only 3 times. We may compute the Banzhaf power index for each member.

Members	Banzhaf Power Index
K, C	$\dfrac{11}{11+11+3+3+3+3}=\dfrac{11}{34}$
W, X, Y, Z	$\dfrac{3}{11+11+3+3+3+3}=\dfrac{3}{34}$